Sample Paper-01
Mathematics
Class – XI

Answers

Section A

1. **Solution:**

$$\frac{3+2i}{1-i}=\frac{3+2i}{1-i}.\frac{1+i}{1-i}$$

$$=\frac{3+2i+3i+2i^2}{1-i^2}=\frac{1}{2}+\frac{5}{2}i$$

$$(1+2i)i-\frac{3+2i}{1-i}=(-2+i)-(\frac{1}{2}+\frac{5}{2}i)=-\frac{5}{2}-\frac{3}{2}i$$

2. **Solution:**

Domain= $[-1,1]$ Range= $[0,\pi]$

3. **Solution :**

$0\le\cos^{-1}x\le\pi$

sin in this interval is positive and hence y is positive

4. **Solution:**

$$\sin^{-1}\left(sin\left(\frac{6\pi}{7}\right)\right)=\sin^{-1}\left(sin\left(\pi-\frac{\pi}{7}\right)\right)$$

$$=\sin^{-1}\left(sin\left(\frac{\pi}{7}\right)\right)$$

$$=-\frac{\pi}{2}\le\frac{\pi}{7}\le\frac{\pi}{2}$$

$$=\frac{\pi}{7}$$

Section B

5. Solution: $(a,2a)$,$(a,-2a)$

6. **Solution:**

$x+7=10$

$x=3$

$x+y=8$

$y=5$

7. **Solution:**

$$y=x^2-x+1$$

$$y=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$$

$$y-\frac{3}{4}=\left(x-\frac{1}{2}\right)^2$$

$$x=\frac{1}{2}+\sqrt{y-\frac{3}{4}}$$

$$f^{-1}(x)=\frac{1}{2}+\sqrt{x-\frac{3}{4}}$$

8. **Solution:**

$Equation\ is\ 8y^2+24x-40y+134=0$

$=4y^2+12x-20y+67=0$

$This\ can\ be\ written\ as$

$y^2-5y=-3x-\frac{67}{4}$

$(y-\frac{5}{2})^2=-3x-\frac{67}{4}+\frac{25}{4}-3(x+\frac{7}{2})$

$Let\ Y=y-\frac{5}{2}$

$X=x+\frac{7}{2}$

$Y^2=-3X$

$This\ is\ of\ the\ form\ y^2=-4ax$

$Latus\ rectum\ is=3$

$Vertex(-\frac{7}{2},\frac{5}{2})$

$Axis\ y=\frac{5}{2}$

$Focus\ (-\frac{7}{2}-\frac{3}{4},\frac{5}{2})$

$Directrix: referred\ to\ New\ axis: X=a=\frac{3}{4}$

$Directrixreferred\ to\ Old\ axis: \frac{3}{4}=x+\frac{7}{2}$

$x=\frac{3}{4}-\frac{7}{2}$

$x=-\frac{11}{4}$

9. **Solution:**

$\frac{7-4i}{3+2i}=\frac{7-4i}{3+2i}\times\frac{3-2i}{3+21}$

$\frac{13-26i}{13}=1-2i$

10. **Solution**

Either both factors are negative or both factors are positive to have this in equality. if $x<2$ both factors are negative and if $x>3$ both factors are positive. Hence the Solution is $x\in\{(-\infty,2)\cup(3,\infty)\}$

11. **Solution**

tan5x = cot2x

$\tan 5x=\tan(\frac{\pi}{2}-2x)$

$$5x=(\frac{\pi}{2}-2x)$$

$$5x=n\pi+(\frac{\pi}{2}-2x)$$

$$7x=n\pi+\frac{\pi}{2}$$

$$x=\frac{1}{7}(n\pi+\frac{\pi}{2})$$

12. **Solution**

Total number of occurrence = $6\times6=36$

On each die there are 3 prime numbers $\{2,3,5\}$

Hence total number of favorable cases $3\times3=9$

Probability of getting a prime in each die = $\frac{9}{36}=\frac{1}{4}$

Section C

13. **Solution:**

$$\phi(\frac{\pi}{12})=\sin 2.(\frac{\pi}{12})$$

$$=\sin\frac{\pi}{6}=\frac{1}{2}$$

$$f(x)=(\frac{1}{2})^3-\frac{1}{2}$$

$$=\frac{1}{8}-\frac{1}{2}=-\frac{3}{8}$$

14. **Solution:**

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$=\frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{m+1}.\frac{1}{2m+1}}=1$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=1$$

$$\tan A+\tan B+\tan A\tan B=1$$

15. **Solution:**

$$f(\sqrt{3})=-1$$

$$f(3)=1$$

$$f(\sqrt{3+1})=1$$

16. **Solution:**

Use the inequality $AM\geq GM$

$$AM\ between\ x,\frac{1}{x}=\frac{x+\frac{1}{x}}{2}$$

$GM \text{ between } x, \frac{1}{x} = \sqrt{x.\frac{1}{x}} = 1$

$\frac{x+\frac{1}{x}}{2} \geq 1$

$x + \frac{1}{x} \geq 2$

$Since \; -1 \leq \sin\theta \leq 1$

$\sin\theta = x + \frac{1}{x} \text{ is impossible}$

17. **Solution:**

$f(x) = \phi(x)$

$f(x) = 3x^2 + 1$

$\phi(x) = 7x - 1$

$3x^2 + 1 = 7x - 1$

$3x^2 - 7x + 2 = 0$

$(x-2)(3x-1) = 0$

$x = 2, x = \frac{1}{3}$

$Hence \; f(x) \text{ and } \phi(x) \text{ are equal when the domain is in the set } \left\{\frac{1}{3}, 2\right\}$

18. **Solution**

$$\lim_{x\to 0}\frac{1-\cos x}{x}$$

$$= \lim_{x\to 0}\frac{1-(1-2\sin^2\frac{x}{2})}{x}$$

$$= \lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{x}$$

$$= \lim_{x\to 0}\frac{\sin\frac{x}{2}}{2\frac{x}{2}}\sin\frac{x}{2}$$

$$= \frac{1}{2}.1.0$$

$$= 0$$

19. **Solution:**

$2\sin^2 x + 14\sin x\cos x + 50\cos^2 x = 26$

$= 2\sin^2 x + 14\sin x\cos x + 50\cos^2 x = 26(\sin^2 x + \cos^2 x)$

$= -24\sin^2 x + 14\sin x\cos x + 24\cos^2 x = 0$

$= 24\sin^2 x - 14\sin x\cos x - 24\cos^2 x = 0$

$= 24\tan^2 x - 14\tan x - 24 = 0$

$$\tan x = \frac{14 \pm \sqrt{196+2304}}{48}$$

$$\tan x = \frac{14 \pm \sqrt{2500}}{48}$$

$$\tan x = \frac{14 \pm 50}{48}$$

$$\tan x = \frac{64}{48} ; or ; -\frac{36}{48}$$

$$\tan x = \frac{4}{3}\ or -\frac{3}{4}$$

20. **Solution**

$$\frac{dy}{dx} = \sin^n x.\{-\sin nx).(n)\} + \cos nx.\{n.\sin^{n-1} x.\cos x\}$$

$$\frac{dy}{dx} = n\sin^{n-1} x(\cos nx.\cos x - \sin x.\sin nx)$$

$$\frac{dy}{dx} = n\sin^{n-1}[\cos(n+1)x]$$

21. **Solution**

$$(a-b)^2 = 5ab$$

$$a^2+b^2-2ab = 5ab$$

$$a^2+b^2 = 7ab$$

$$(a+b)^2 = 9ab$$

$$a+b = 3\sqrt{ab}$$

$$\frac{1}{3}(a+b) = \sqrt{ab}$$

$$\log\left(\frac{1}{3}(a+b)\right) = \frac{1}{2}(\log a + \log b)$$

22. **Solution**

First term = $\frac{1}{10}$

Second term= $\frac{7}{10^2}$

Third term= $\frac{7^2}{10^3}$

r = $\frac{7}{10}$

This is a GP

Sum to infinity= $\dfrac{\frac{1}{10}}{1-\frac{7}{10}}$ = $\frac{1}{3}$

23. **Solution**

Mean= 14= $\dfrac{8+12+13+15+22+14}{6}$

x_i	$x_i - Mean$	$(x_i - Mean)^2$
8	-6	36
12	-2	4
13	-1	1
15	1	1
22	8	64
14	0	0
		$\Sigma(x_i - Mean)^2 = 106$

Variance= $\dfrac{1}{n}\Sigma(x_i - Mean)^2 = \dfrac{106}{6} = 17.66$

SD= $\sqrt{Variance} = \sqrt{17.66} = 4.2$

Section D

24. **Solution**

$$Let\ \frac{a}{y} = x$$

$$by = \frac{ab}{x}$$

$$f(1+\frac{a}{y})^{by} = f\left[(1+x)^{\frac{1}{x}}\right]^{ab}$$

25. **Solution:**

Probability of all the three hitting the target= $\dfrac{4}{5}.\dfrac{3}{4}.\dfrac{2}{3} = \dfrac{2}{5}$

Probability of A alone missing the target = $\dfrac{1}{5}.\dfrac{3}{4}.\dfrac{2}{3} = \dfrac{1}{10}$

Probability of B alone missing the target = $\dfrac{4}{5}.\dfrac{1}{4}.\dfrac{2}{3} = \dfrac{2}{15}$

Probability of C alone missing the target = $\dfrac{4}{5}.\dfrac{3}{4}.\dfrac{1}{3} = \dfrac{1}{5}$

The probability that the target being hit at least two= $\dfrac{2}{5} + \dfrac{1}{10} + \dfrac{2}{15} + \dfrac{1}{5} = \dfrac{5}{6}$

26. **Solution**

Let T_{r+1} be the term that is independent of x

Then,

$$T_{r+1} = {}^9C_r (ax^2)^r (-\frac{b}{x})^{9-r}$$

$$2r + (r-9) = 0$$

$$r = 3$$

4th term is independent of x

$$T_4 = {}^9C_3 (a)^3 (-b)^6$$

$$= {}^9C_3 (a)^3 (b)^6$$